Global Dynamics of a Mathematical Model on Smoking with Media Campaigns

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Abstract:

In this paper, a non-linear model is proposed to study the effect of media campaigns on the smoking cessation. In our model we found two equilibria one of them is the smoking-free equilibrium (SFE) and the other corresponds to the presence of smoking and their stability analyzed by the theory of differential equation and computer simulation. The model study shows that the impact of awareness created by media campaigns on the smoking cessation increases, smokers population decreases. Numerical simulation also supports the obtained results.

Keywords: compartmental model, smoking, media campaigns, stability, numerical simulation.

AMS Classification: 34D20 , 34D23.

1. Introduction:

The smoking subject is an interesting area to study. There is strong medical evidence that smoking tobacco is related to more than two dozen diseases and conditions. Cigarette smoking and exposure to tobacco smoke causes is one of the biggest public health threats that kills more that 5 million people every year. Estimate revels that if current trends continue, this figure may increase to more than eight million people per year by 2030[1]. The reason for that high number is that Tobacco smoke use is a major cause of many of the world's top killer diseases including cardiovascular disease, chronic lung disease and lung cancer. Smoking is often the hidden cause of many killing disease [2]. It is the single most preventable cause of disease, disability and death in many countries. Mostly youngsters are falling prey to the menace of smoking. Now in these days, cigarette smoking has become a common as well as fashionable trend among youngsters. The smoking rate among children with three or more friends who smoking is ten times higher than the rate among those who report that none of their friends smoke [3].

Further the tobacco advertising, promotion and sponsorship are also banned in nineteen countries which comprise almost six percent of world's population [1]. Government of India has also passed a law enforcing ban on smoking at public places with effect from two October, 2008 [4, 5]. The increasing focus of government and other health care organization on prevention and control and addictive behaviors, such as tobacco and alcohol use have also reckoned interest of mathematical modelers in this area [6-12]. The WHO Tobacco free Initiative is raising awareness about global tobacco epidemic and preventive steps requisite to reduce tobacco use [13]. Awareness programs by media and behavioral interventions have an enormous effect on the future course of and epidemic [14-18]. Therefore to asses the impact of media campaigns on smoking cessation, we formulate and analyze a mathematical model for the spread of awareness amongst population in smoking age.

The rest of paper in organized as follows: In section 2, a mathematical model to assess the impact of media campaigns on prevalence of smoking in the society. In the next section 3, equilibria of the proposed model are obtained and their stability discussed. In section 4, the global stability of endemic equilibrium point discussed. The numerical simulation confirming the analytically obtained results is given in section 5. Finally, we end the paper with a brief conclusion.

2. Mathematical Model:

Let the total population size at time t be denoted by $T(t)$. We divide the population $T(t)$ into four subclasses potential smokes (nonsmokers) $P(t)$, smokers $S(t)$, smokers who temporarily quit smoking $Q_t(t)$, and smokers who permanently quit smoking $Q_p(t)$, such that $T(t) = P(t) + S(t) + Q_t(t) + Q_p(t)$. The number of media campaigns to promote smoking cessation in the region under consideration are M. We describe the dynamics of smoking model with the impact of awareness created by media campaign on the smoking cessation is given by the following system of nonlinear ordinary differential equations:

$$\frac{dP}{dt} = \mu - \mu P - \beta PS,$$

$$\frac{dS}{dt} = \beta PS + \alpha SQ_t - (\mu + \gamma)S - \eta SM,$$

$$\frac{dQ_t}{dt} = \eta SM - \alpha SQ_t + \gamma(1-\sigma)S - \mu Q_t - \varepsilon Q_t, \tag{1}$$

$$\frac{dQ_p}{dt} = \sigma\gamma S - \mu Q_p + \varepsilon Q_p,$$

$$\frac{dM}{dt} = \phi S - \phi_0 (M - M_0).$$

We assume that the mortality is balanced with the influx of people and so the total population constant. It is considered that non-smokers start smoking due to peer influence and the join the smokers class with rate βPS. On the other hand some of the smokers become aware of ill-effects of smoking due to media campaigns, quit smoking and join the quitter class with a rate ηSM. In the above system, μ is the rate of natural death which are assumed identical, α is the contact rate between smokers and temporary quitters who revert back to smoking, γ is the rate of quitting smoking, $(1-\sigma)$ is the fraction of smokers who temporarily quit smoking (at a rate γ), σ is the remaining fraction of smokers who permanently quit smoking (at a rate γ). The constant β is effective contact rate, and it is product of the average number of influential contacts per unit time made by a non-smoker and probability of becoming smoker following a contact, with a smoker. The media campaigns disseminate awareness among smokers at a rate η and ε is the rate at which temporary quitters quit smoking permanently. Further, it is considered that some media campaigns fade or lose their impact on people, but a baseline number of media campaigns M_0 are maintained in the system. Moreover, ϕ and ϕ_0 are constants denoting rate of implementing and fading of media campaigns, respectively.

In model (1), the total population is constant and $P(t), S(t), Q_t(t), Q_p(t)$ are respectively the proportions of potential smokers, smokers, temporarily quitters and permanent quitters where $P(t)+S(t)+Q_t(t)+Q_p(t)=1$. Since the variable Q_p, of model (1) does not appear in the first three equations, we will consider the subsystem:

$$\frac{dP}{dt} = \mu - \mu P - \beta PS,$$

$$\frac{dS}{dt} = \beta PS + \alpha SQ_t - (\mu+\gamma)S - \eta SM,$$

$$\frac{dQ_t}{dt} = \eta SM - \alpha SQ_t + \gamma(1-\sigma)S - \mu Q_t - \varepsilon Q_t, \tag{2}$$

$$\frac{dM}{dt} = \phi S - \phi_0 (M - M_0).$$

Where $P(0)=P_0>0,\ S(0)=S_0>0,\ Q_t(0)=Q_{t0}\geq 0$ and $M(0)=M_0>0.$

Clearly, $\frac{dP}{dt}+\frac{dS}{dt}+\frac{dQ_t}{dt} \leq \mu - \mu(P+S+Q_t)$. Therefore, as in [12] it can shown that $\underset{t\to\infty}{\text{limit}}\ Sup T(t) \leq 1$. Since the model system (1) monitors human population, all the variables and parameters are assumed to be non-negative for all $t \geq 0$. Thus we study the model (2) in the positively invariant set,

$$\Omega = \left\{ (P, S, Q_t, M) \in R^4{}_+ :\ P + S + Q_t \leq 1,\ 0 \leq M \leq \frac{\phi}{\phi_0} = M_R \right\},$$

which is region of attraction for the system.

3. Equilibria and their stability analysis:

3.1 Equilibria:

The model system (2) always has a smoking-free equilibrium (SFE) $E_0(1,0,0,M_0)$, in which whole population is non-smoker. Also, the model system (2) exhibits an Endemic equilibrium $E^*(P^*, S^*, Q^*{}_t, M^*)$.

By setting $dP/dt=0,\ dQ_t/dt=0,\ dM/dt=0,$ all components of E^* can be expressed in terms of S^* as,

$$M^* = M_0 + \frac{\phi}{\phi_0} S^*,\quad P^* = \frac{\mu}{\mu+\beta S^*},\quad Q^*{}_t = \frac{\left(\eta S^* M_0 + \eta \frac{\phi}{\phi_0} S^{*2} + \gamma(1-\sigma) S^*\right)}{(\alpha S^* + \mu + \varepsilon)} \tag{3}$$

Substituting these values in the following equation,

$\beta P + \alpha Q_t - \eta M - (\mu + \gamma) = 0.$

On simple calculations, this gives a quadratic equation in S^* as,

$$A_1 S^{*2} + A_2 S^* + A_3 = 0. \tag{4}$$

Where,

$$A_1 = \gamma(1-\sigma)\beta\alpha - \eta\beta(\mu+\varepsilon)\frac{\phi}{\phi_0} - \beta\alpha(\mu+\gamma),$$

$$A_2 = \gamma(1-\sigma)\alpha\mu + (\beta-\mu-\gamma)\alpha\mu - \beta(\mu+\varepsilon)(\eta M_0 + \mu + \gamma) - \eta\mu(\mu+\varepsilon)\frac{\phi}{\phi_0}, \text{ and}$$

$$A_3 = \mu(\mu+\varepsilon)(\eta M_0 + \mu + \gamma)(R-1).$$

It is clear that sign of A_1 and A_2 can be different for different parameter values. The sign of A_3 depends of $(R-1)$, where $R = \dfrac{\beta}{(\eta M_0 + \mu + \gamma)}$.

Let $S_{\pm}^*$ denote the roots of equation (4), then

$$S_{\pm}^* = \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1} \tag{5}$$

We now consider three cases:

Case1: If $R > 1$ so that $A_3 > 0$. In this case S_-^* is always negative and S_+^* is always positive. It follows that equation (4) has a unique positive solution and there exists a unique positive equilibrium E^* whenever $R > 1$.

Case2: If $R < 1$ so that $A_3 < 0$. Here equation (4) has both the root (i.e., S_-^* and S_+^*) positive if $A_2 < 0$ and $A_2^2 - 4A_1A_3 > 0$ otherwise none. This implies that, multiple endemic equilibria (say E_-^* and E_+^*) may exist when $R < 1$.

Case3: If $R = 1$ so that $A_3 = 0$ and $A_1 > 0,\ A_2 > 0$ there are no positive solutions.

3.2 Reproduction number:

The quantity R plays the role analogous to the basic reproduction number in epidemic models. In the mathematical epidemiology the basic reproduction number R_0 defined as the expected number of secondary infections arising from a single individual during his or her entire infectious period, in a population of susceptible [19]. This concept is fundamental to the study of epidemiology and within-host pathogen dynamics. Most importantly, R_0 often serves as a threshold parameter that predicts whether an infection will spread. Related parameters which share this threshold behavior, hoowever, may or may not give the true value of R_0. A fundamental result in mathematical epidemiology, which most epidemic model follow, is that endemic equilibrium exists whenever $R_0 > 1$, which physically means that the disease can invade the population if each infective generates, on average, more than one new infective cases in the population. Similarly, on analyzing the proposed model, we found that endemic equilibrium exists whenever the quantity R greater than unity. We write it as $R = \dfrac{\beta}{(\eta M_0 + \mu + \gamma)}$. Note that a single smoker creates β new smokers in unit time and average time spent in the smoker class is $1/(\eta M_0 + \mu + \gamma)$.

3.3 Stability analysis:

Now we proceed to study the stability behavior of equilibria E_0 and E^*. The Jacobian matrix at Smoking-Free Equilibrium (SFE) is given by

$$J_{E_0} = \begin{bmatrix} -\mu & -\beta & 0 & 0 \\ 0 & -(\mu+\gamma) - \eta M_0 + \beta & 0 & 0 \\ 0 & \gamma(1-\sigma) + \eta M_0 & -\mu - \varepsilon & 0 \\ 0 & \phi & 0 & -\phi_0 \end{bmatrix}.$$

It is apparent that eigenvalues of J_{E_0} is $-\phi_0, -\mu-\varepsilon, -\mu,$ and $\beta - \eta M_0 - (\mu+\gamma)$. It is noted that here that if $R = \dfrac{\beta}{(\eta M_0 + \mu + \gamma)} < 1$ all the eigenvalues of J_{E_0} are negative whereas one eigenvalue becomes positive if $R > 1$. Hence we make an assertion that SFE is locally asymptotically stable if $R < 1$ and unstable (saddle point) if $R > 1$.

Now for the endemic equilibrium the Jacobian matrix is given as,

$$J_{E^*} = \begin{bmatrix} -\mu - \beta S^* - \lambda & -\beta P^* & 0 & 0 \\ \beta S^* & -(\mu+\gamma) - \eta M^* + \beta P^* + \alpha Q_t^* - \lambda & \alpha S^* & -\eta S^* \\ 0 & -\alpha Q_t^* + \gamma(1-\sigma) + \eta M^* & -\alpha S^* - \mu - \varepsilon - \lambda & \eta S^* \\ 0 & \phi & 0 & -\phi_0 - \lambda \end{bmatrix}$$

The characteristic polynomial for J_{E^*} is given by

$$\lambda^4 + b_3\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0 = 0 \tag{6}$$

where,

$$b_0 = (\alpha S^* + \mu + \varepsilon)\phi_0 \beta^2 P^* S^* - (\mu + \beta S^*)\begin{Bmatrix} \alpha S^* \phi_0 \left(\gamma(1-\sigma) + \eta M^* - \alpha Q_t^*\right) + \alpha\phi\eta S^{*2} - \eta S^* \phi(\alpha S^* + \mu + \varepsilon) \\ + \phi_0(\alpha S^* + \varepsilon + \mu)(\beta P^* + \alpha Q_t^* - \mu - \gamma - \eta M^*) \end{Bmatrix}$$

$$b_1 = (\phi_0 + \alpha S^* + \mu + \varepsilon)\beta^2 P^* S^* - \begin{Bmatrix} \alpha S^* \phi_0 \left(\gamma(1-\sigma) + \eta M^* - \alpha Q_t^*\right) + \alpha\phi\eta S^{*2} - \eta S^* \phi(\alpha S^* + \mu + \varepsilon) \\ + \phi_0(\alpha S^* + \varepsilon + \mu)(\beta P^* + \alpha Q_t^* - \mu - \gamma - \eta M^*) \end{Bmatrix} -$$
$$\left\{(\beta P^* + \alpha Q_t^* - \mu - \gamma - \eta M^*)(\phi_0 + \alpha S^* + \mu + \varepsilon) - \phi_0(\alpha S^* + \mu + \varepsilon) + \alpha S^*\left(\gamma(1-\sigma) + \eta M^* - \alpha Q_t^*\right) - \eta S^* \phi\right\}$$
$$\left(\mu + \beta P^*\right)$$

$$b_2 = (\phi_0 + \alpha S^* + \mu + \varepsilon)(\mu + \beta S^*) + \begin{Bmatrix} (\beta P^* + \alpha Q_t^* - \mu - \gamma - \eta M^*)(\phi_0 + \alpha S^* + \mu + \varepsilon) - \\ \phi_0(\alpha S^* + \mu + \varepsilon) + \alpha S^*\left(\gamma(1-\sigma) + \eta M^* - \alpha Q_t^*\right) - \eta S^* \phi \end{Bmatrix} + \beta^2 P^* S^*$$

$$b_3 = (\mu + \beta S^*) + (\phi_0 + \alpha S^* + \mu + \varepsilon) - (\beta P^* + \alpha Q_t^* - \mu - \gamma - \eta M^*)$$

Now by using *Routh-Hurwitz Criterion,* we can say that endemic equilibrium E^* is locally asymptotically stable if and only if the following inequalities satisfied: $b_3 b_2 > b_1$ and $b_3 b_2 b_1 > (b_1^2 + b_3^2 b_0)$.

On the basis of our study regarding the equilibria of model system (2) and their stability we have following result:

Theorem 1. (i) If $R < 1$, then the smoking free equilibrium (SFE) E_0 is locally asymptotically stable.

(ii) If $R > 1$, then smoking free equilibrium (SFE) E_0 becomes unstable and the endemic equilibrium E^* exists, which is locally asymptotically stable if the following inequalities hold: $b_3 b_2 > b_1$ and $b_3 b_2 b_1 > (b_1^2 + b_3^2 b_0)$.

4. Global Stability:

Consider the following Lyapunov function as,

$$V = \frac{1}{2}(P - P^*)^2 + \frac{1}{2}q_1(S - S^*)^2 + \frac{1}{2}q_2(Q_t - Q_t^*)^2 + \frac{1}{2}q_3(M - M^*)^2,$$

where, q_1, q_2 and q_3 are some positive constants to be chosen later on. On differentiating V with respect to t along the solutions of model system (2), we get

$$\dot{V} = -a_{11}(P - P^*)^2 + a_{12}(P - P^*)(S - S^*) - \frac{1}{3}a_{22}(S - S^*)^2$$
$$-\frac{1}{3}a_{22}(S - S^*)^2 + a_{23}(S - S^*)(Q_t - Q_t^*) - \frac{1}{2}a_{33}(Q_t - Q_t^*)^2$$

$$-\frac{1}{3}a_{22}(S-S^*)^2+a_{24}(S-S^*)(M-M^*)-\frac{1}{2}a_{44}(M-M^*)^2.$$

$$-\frac{1}{2}a_{33}(Q_t-Q_t^*)^2+a_{34}(Q_t-Q_t^*)(M-M^*)-\frac{1}{2}a_{44}(M-M^*)^2$$

We choose $q_1=q_2=q_3=1$ and sufficient condition for $\dot{V}$ to be negative definite are that the following inequalities satisfied:

$$a_{12}^2<\frac{4}{3}a_{11}a_{22}$$

$$a_{23}^2<\frac{2}{3}a_{22}a_{33}$$

$$a_{24}^2<\frac{2}{3}a_{22}a_{33}$$

$$a_{34}^2<a_{33}a_{44.}$$

where, $a_{11}=(\mu+\beta S)$, $a_{22}=q_1(\eta M+\mu+\gamma-\beta P-\alpha Q_t)$, $a_{33}=q_2(\mu+\varepsilon)$, $a_{44}=q_3\phi$,
$a_{12}=(q_1\beta S^*-\beta P^*)$, $a_{23}=(q_1\alpha S^*+q_2\eta M^*+q_2\gamma(1-\sigma))$, $a_{34}=q_2\eta S$, $a_{24}=(q_3\phi-q_1\eta S)$.

This shows that V is a Liapunov's function for the model system (2), implying that endemic equilibrium E^* is globally asymptotically stable.

On the basis of our study regarding the endemic equilibrium E^* of model system (2) and their global stability we have following result:

Theorem 2. The endemic equilibrium $E^*(P^*,S^*,Q_t^*,M^*)$ if exists $R>1$ is globally or nonlinearly asymptotically stable in the region Ω and we choose $q_1=q_2=q_3=1$ if the following inequalities satisfied:

$$(q_1\beta S^*-\beta P^*)^2<\frac{4}{3}(\mu+\beta S)\,q_1(\eta M+\mu+\gamma-\beta P-\alpha Q_t),$$

$$(q_1\alpha S^*+q_2\eta M^*+q_2\gamma(1-\sigma))^2<\frac{2}{3}q_1(\eta M+\mu+\gamma-\beta P-\alpha Q_t)q_2(\mu+\varepsilon),$$

$$(q_3\phi-q_1\eta S)^2<\frac{2}{3}q_1(\eta M+\mu+\gamma-\beta P-\alpha Q_t)q_2(\mu+\varepsilon),$$

$$(q_2\eta S)^2<q_2(\mu+\varepsilon)q_3\phi.$$

5. Numerical simulation:

In this section, we have done the numerical simulation for model system (2) using MATLAB 7.5.0. The parameter values used for numerical simulation are,

$\mu = 0.12, \beta = 0.46, \alpha = 0.34, \gamma = 0.03, \eta = 0.042, \sigma = 0.4, \varepsilon = 0.23, \phi = 0.3, \phi_0 = 0.01, M_0 = 3.$
The equilibrium components are found as follows:

$P^* = 0.7733,\ S^* = 0.0765,\ Q_t^* = 0.0489,\ M^* = 5.2940.$

For the above set of parameters it may be checked that the conditions of existence of endemic equilibrium E^* and the local stability and nonlinear or global stability conditions are satisfied and also the set of above parameter values, the reproduction number R is 1.6667.The Figures 1 and 2 shows the global stability in $P-Q_t$ and $P-S$ space respectively. It is apparent from these figure that all the trajectories in the $P-Q_t$ and $P-S$ space are approaching towards endemic equilibrium E^*.
Further in Figures 3 and 4 we have shown variation of smokers population $S(t)$ with respect to time $'t'$ for different values of η and M_0 respectively. From these figures it may be noted that number of smokers population decreases as the effective contact rate between smokers population and media campaigns and baseline number of media campaigns increases respectively. In Figure 5 and 6 we have also shown, the variation of smokers population $S(t)$ with respect to time $'t'$ for different values of ϕ and β respectively. From this figure it is noted that the contact rate between non-smokers and smokers increases and the rate of implementing awareness program increases then smokers population increases and decreases respectively. Also in figures 7 and 8 we have shown that rate of quitting smoking γ and the effective contact rate α between smokers and temporary quitters who revert back to smoking increases, smokers population $S(t)$ decreases and increases respectively.

6. Conclusion:
The present study comprises a non-linear mathematical model to assess the impact of media on smoking cessation. The model exhibits two non negative equilibrium namely, smoking-free equilibrium (SFE) and endemic equilibrium. A threshold R, analogous to basic reproduction number in epidemic models is obtained. If its value is less than unity, SFE is locally asymptotically stable. But if the value of reproduction number is greater than unity, SFE becomes unstable and endemic equilibrium exists. By using Routh-Hurwitz Criteria shows that endemic equilibrium is locally asymptotically stable. Also a Liapunov function is used to shown global stability of endemic equilibrium under certain conditions are satisfied. The media must devise integrated campaigns that focus on motivating people to quit smoking and also make them acquainted with the practices that can prevent relapse. Moreover, some media campaigns propagating awareness must be maintained in the system. The sustained media campaigns that aware people to quit smoking and overcome the cravings of smoking after quitting can cease the smoking epidemic.

Figures:

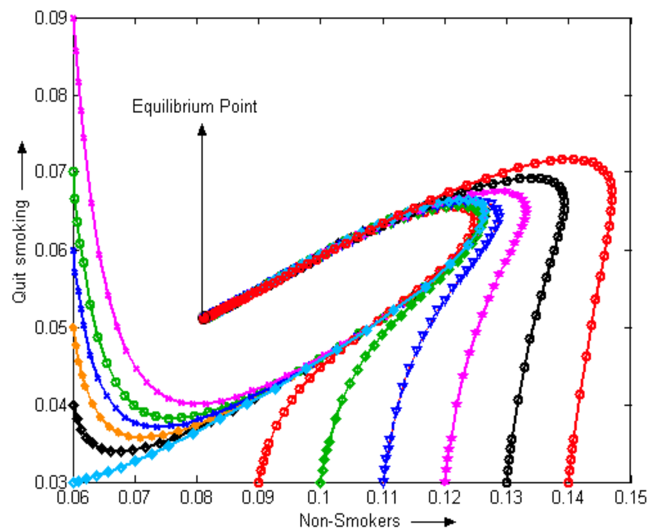

Fig.1 Global stability in $P-Q_t$ space.

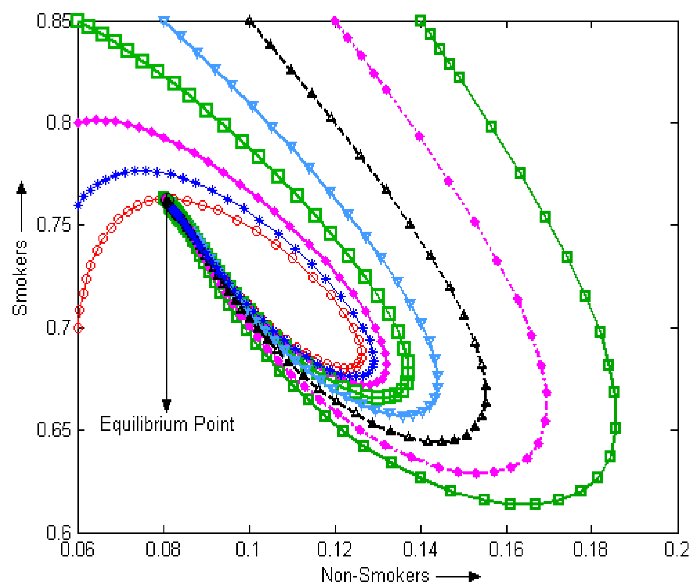

Fig.2 Global stability in $P-S$ space.

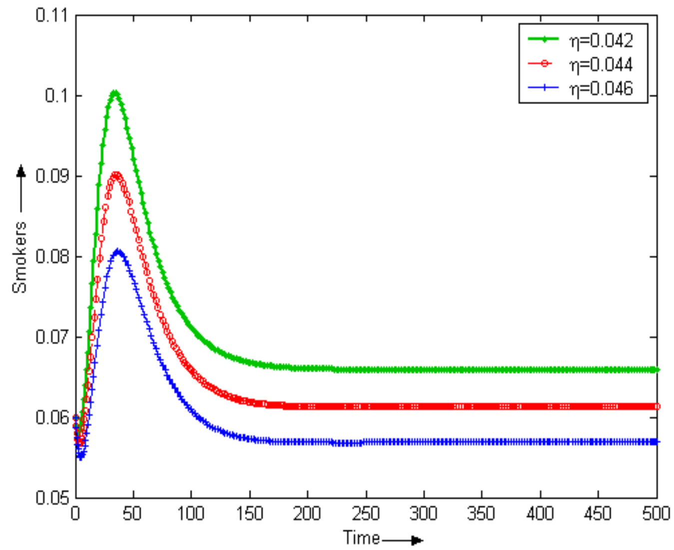

Fig. 3 Variation of smokers population $S(t)$ with respect to time $'t'$ for different values of η.

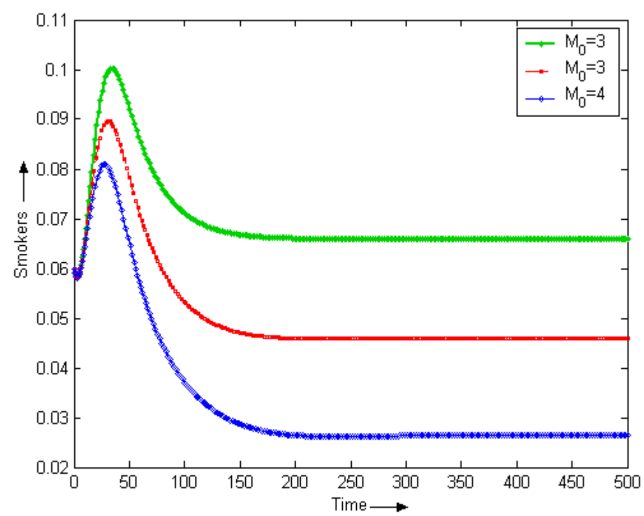

Fig. 4 Variation of smokers population $S(t)$ with respect to time $'t'$ for different values of M_0.

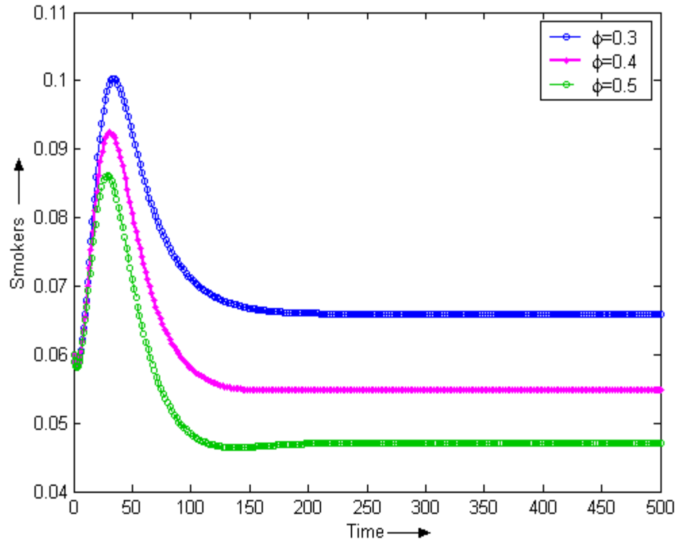

Fig. 5 Variation of smokers population $S(t)$ with respect to time $'t'$ for different values of ϕ.

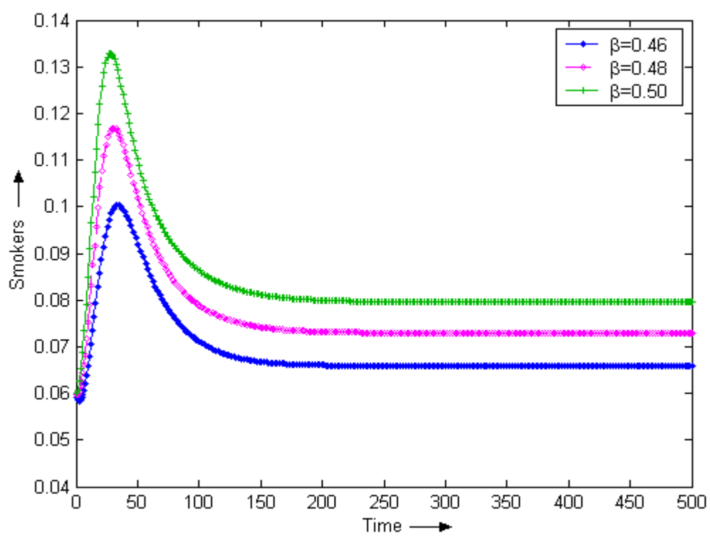

Fig. 6 Variation of smokers population $S(t)$ with respect to time $'t'$ for different values of β.

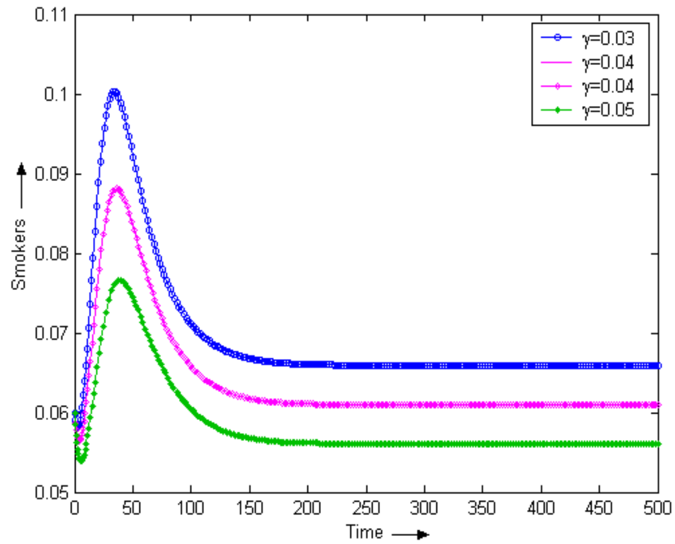

Fig. 7 Variation of smokers population $S(t)$ with respect to time $'t'$ for different values of γ.

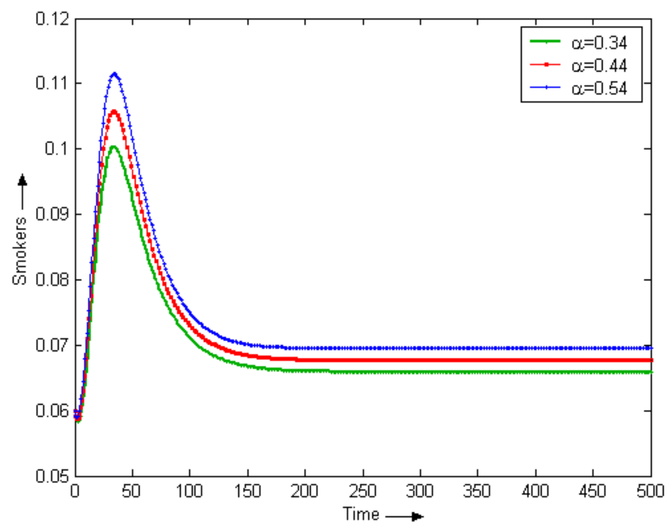

Fig. 8 Variation of smokers population $S(t)$ with respect to time $'t'$ for different values of α.

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